

# Multi-objective optimization for hybrid fuel cells power system under uncertainty

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## Abstract

One of the major applications of fuel cells is as onsite stationary electric power plants. Several types of configurations have been hypothesized and tested for these kinds of applications at the conceptual level but hybrid power plants are one of the most efficient. These are designs that combine a fuel cell cycle with other thermodynamic cycles to provide higher efficiency. Generally, the heat rejected by the fuel cell at a higher temperature is used in a bottoming cycle to generate steam. In this work we are considering a conceptual design of a solid oxide fuel cell-proton exchange membrane (SOFC-PEM) fuel cell hybrid power plant [R. Geisbrecht, Compact Electrochemical Reformer Based on SOFC Technology, AIChE Spring National Meeting, Atlanta, GA, 2000] in which the high temperature SOFC fuel cell acts both as electricity producer and fuel reformer for the low temperature PEM fuel cell (PEMFC). The exhaust from the PEM fuel cell goes to a waste hydrogen burner and heat recovery steam generator that produces steam for further utilizations. Optimizing this conceptual design involves consideration of a number of objectives. The process should have low pollutant emissions as well as cost competitive with the existing technology. The solution of a multi-objective optimization problem is not a single solution but a complete non-dominated or Pareto set, which includes the alternatives representing potential compromise solutions among the objectives. This makes a range of choice available to decision makers and provides them with the trade-off information among the multiple objectives effectively. This paper presents the optimal trade-off design solutions or the Pareto set for this hybrid power plant through a multi-objective optimization framework. This hybrid technology is new and the system level models used for fuel cells performance have significant uncertainties in them. In this paper, we characterize these uncertainties and study the effect of these uncertainties on the optimal trade-offs. The framework developed in this work forms the basis for optimal design and synthesis of any power plant under uncertainties in the face of multiple objectives.

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## 1. Introduction

Fuel cells are an important technology for a potentially wide variety of applications including micropower, auxiliary power, transportation power, stationary power for buildings and other distributed generation applications, and central power. These applications are prevalent in a large number of industries worldwide. Research efforts are being made to design fuel cell plants with low emissions, high efficiencies, high performance and low costs. High efficiency hybrid fuel cell power plants are being conceptualized in order to meet the great expectations from this technology. Also, a large number of materials need to be considered in the fuel cells for electrolyte issues, electrode performance

issues, and for different configurations in order to obtain the desired properties. Moreover, the development in the area of new materials and other technologies where the performance and economic data is scarce and/or incomplete, calls for consideration of uncertainties in the design and optimization. Furthermore, the problem involves multiple objectives such as cost, process efficiency and pollutant emissions. The conflicting nature of these objectives makes the handling of this problem an even more formidable task. The solution of a multi-objective problem is a complete non-dominated or Pareto set, which includes the alternatives representing potential compromise solutions among the objectives. This makes a range of choice available to decision makers and provides them with the trade-off information among the multiple objectives effectively. In this paper, we optimize the cost and performance of a new solid oxide fuel cell-proton exchange membrane (SOFC-PEM) hybrid

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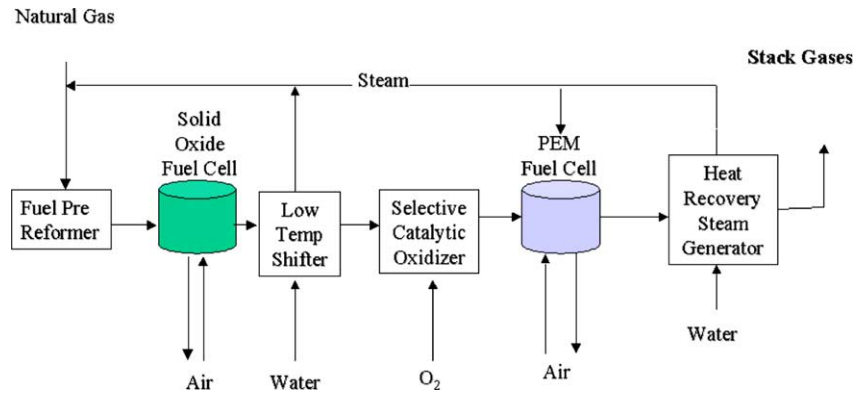


Fig. 1. SOFC-PEM hybrid fuel cell power plant.

power plant, evaluate and analyze the trade-offs between the multiple objectives using an efficient multi-objective framework. We use available experimental data from literature to characterize, quantify uncertainties in the current density characteristics of the fuel cell models. The uncertainties are propagated through the flowsheet model and the optimization framework to obtain stochastic trade-off surfaces.

The paper has been divided into six sections. In Section 2, we discuss the salient features of the new SOFC-PEM hybrid power plant. In Section 3, the concept of multi-objective optimization and a novel multi-objective optimization framework based on a new and efficient algorithm MINSOOP for solving nonlinear multi-objective programming (MOP) is described. The results of the MOP problem are presented in Section 4. Section 5 presents the stochastic approach to MOP. The deterministic and stochastic results have been compared in this section. Section 6 puts forth conclusions drawn from this work.

## 2. SOFC-PEM hybrid power plant

A solid oxide fuel cell-proton exchange membrane hybrid power plant was chosen as our case study. This is a conceptual design being the focus of current research and development program at National Energy Technology Laboratories (NETL) [1]. As shown in Fig. 1, this plant contains two fuel cells combined with a heat recovery steam

Table 1  
Base case design and performance of the SOFC-PEM hybrid fuel cell power plant

	SOFC	PEM
Temperature (°F)	1750	176
Pressure (psi)	20	25
Current density (mA/cm <sup>2</sup> )	75	190
Fuel utilization (%)	70	
Equivalence ratio	1.25	

Power rating: 1472 kW; overall efficiency: 72.6%; capital cost: US\$ 1773/kW; CO<sub>2</sub> emissions: 5.61 kg/kWh; cost of electricity: 6.35 c/kWh.

generation cycle. This use of two fuel cells makes the cycle up to 37.8% more efficient than the case where only SOFC is used (maximum efficiency of 52.4%). Natural gas, which is used as the fuel, is processed in a fuel pre reformer and fed to the SOFC, which acts as both electricity producer as well as a fuel reformer for the PEM fuel cell (PEMFC). The exhaust fuel from the SOFC is cooled and shifted in a low temperature shifter that also functions as a low-pressure steam boiler. Shifted fuel gas is then treated with pure oxygen in a selective catalytic oxidizer to reduce CO from several hundred parts per million (ppm) to below 10 ppm. The utilization of this reformed fuel is completed in the PEMFC where more favorable thermodynamics apply. The exhaust from the PEMFC goes to a waste hydrogen burner and heat recovery steam generator to utilize the waste heat of the exhaust stream to make steam from water. This steam produced in the low temperature shifter and the heat recovery steam generator is used in both the SOFC and PEMFC. In the SOFC, steam is used as a reactant for the reforming and downstream shift reactions and to control against the carbon. In the PEMFC, steam is used to humidify fuel and air streams to maintain water balance in the electrolyte and

Table 2  
The objectives, constraints and decision variables for the hybrid fuel cell power plant

Objectives:  
 Min capital cost (CAP)  
 Min cost of electricity (COE)  
 Min CO<sub>2</sub> emissions (CO2EM)  
 Max current density SOFC (CDSOFC)  
 Max current density PEM (CDPEM)  
 Max overall efficiency (ACEFF)

Subject to:  
 Mass and energy balance constraints  
 Power rating of 1472 kW (base case)

Decision variables:  
 Fuel utilization (UTIL)  
 Equivalence ratio (ERAT)  
 Pressure of the PEM (PPEM)  
 Fuel flow (FUEL)  
 Air flow (AIR)

electrodes. Design and performance results obtained using ASPEN simulator model [1] are summarized in Table 1.

The key assumptions in this advanced technology model are: (1) staged cells can be manufactured and installed at the same cost as unstaged cells, and (2) a sufficient number of cells can be staged so as to closely approach the limiting case performance for staged cells. As shown in the base case table (Table 1), there are six objectives to be simultaneously optimized subject to highly complex non-linear models. Moreover some objectives are conflicting and incommensurable, i.e. it is not possible to improve them simultaneously and there is always a trade-off. Hence a multi-objective optimization framework is required to obtain the Pareto set. Table 2 shows the multi-objective optimization problem formulation for the current study of the hybrid power plant.

### 3. Multi-objective optimization framework

As mentioned at the end of the previous section, the economic objectives along with CO<sub>2</sub> emissions, overall system efficiency and fuel cell current densities form the set of multiple objectives to be optimized simultaneously for which multi-objective optimization is required [2,3]. Multi-objective problems appear virtually in every field and in a wide variety of contexts [4,5]. The problems solved vary from designing spacecrafts, aircraft control systems, bridges, vehicles, and highly accurate focusing systems, to forecasting manpower supplies, selecting portfolios, blending sausages, planning manufacturing systems, managing nuclear waste disposal and storage, allocating water resources, and solving pollution control and management problems.

#### 3.1. The MINSOOP algorithm

A generalized multi-objective optimization (or multi-objective programming) problem can be formulated as follows:

$$\begin{aligned} &\text{minimize } Z = Z_i, \quad i = 1, \dots, p; \quad p \geq 2 \\ &\text{Subject to} \\ &h(x, y) = 0, \\ &g(x, y) \leq 0, \end{aligned} \quad (1)$$

where  $x$  and  $y$  are continuous and discrete decision variables, and  $p$  is the number of objective functions. The functions  $h(x, y)$  and  $g(x, y)$  represent equality and inequality constraints, respectively. Though there is a large array of analytical techniques to solve this MOP problem, they are generally divided into two basic types: preference-based methods and generating methods [5]. Preference-based method like goal programming attempt to quantify the decision maker's preference and with this information, identifies the solution that best satisfies the decision maker's preference. Generating methods, such as the weighting method and the constraint method, have been developed to find the exact Pareto set or

an approximation of it. As is well known, mathematics cannot isolate a unique optimum when there are multiple competing objectives. Mathematics can at most aid designers to eliminate design alternatives dominated by others, leaving a number of alternatives in what is called the Pareto set [6]. For each of these designs, it is impossible to improve one objective without sacrificing the value of another relative to some other design alternative in the set. From among the dominating solutions, it is then a value judgment by the decision maker to select which design is the most appropriate. At issue is an effective means to find the members of the Pareto set for a design problem, especially when there are more than two or three objectives; the analysis per design requires significant computations to complete, and there are an almost uncountable number of design alternatives. A pure algorithmic approach to solving is to pick one of the objectives to minimize while the remaining others are turned into inequality constraints with a parametric right-hand-side,  $L_k$ . The problem takes on the following form:

$$\begin{aligned} &\text{minimize } Z_j, \\ &\text{Subject to} \\ &h(x, y) = 0, \\ &g(x, y) \leq 0, \\ &Z_k \leq L_k, \quad k = 1, \dots, j-1, j+1, \dots, p; \quad p \geq 2 \end{aligned} \quad (2)$$

where  $Z_j$  is the chosen  $j$ th objective that wished to be optimized. Solving repeatedly for different values of  $L_k$  chosen between the upper,  $Z_L(j)$  and lower bounds  $Z_U(j)$  leads to the Pareto set. The multi-objective optimization algorithm, minimization of single objective optimization problems (MINSOOP) [7,8] used in this work uses the Hammerley sequence sampling [9,10] to generate combinations of the right-hand-side. MINSOOP exploits the  $n$ -dimensional uniformity of the HSS technique to obtain greater efficiency. Fig. 2 shows how this MINSOOP algorithm improves efficiency for a simple, nonlinear, convex optimization problem, as the number of objectives increase.

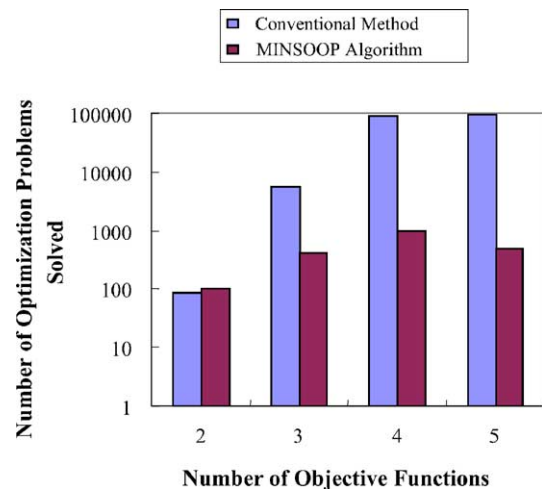


Fig. 2. Conventional method vs. the new efficient MINSOOP algorithm.

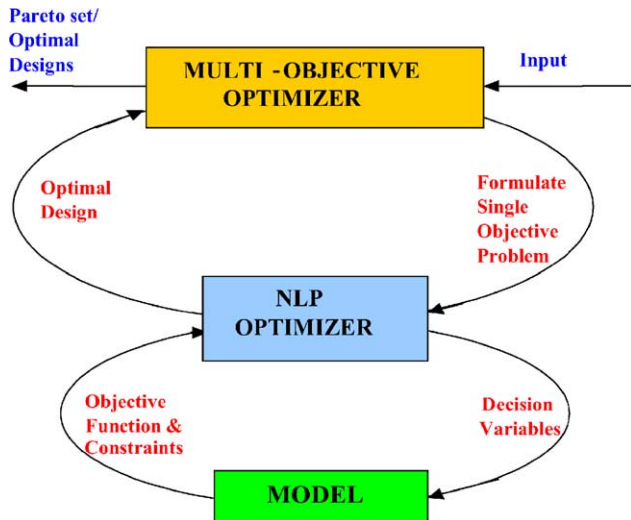


Fig. 3. Deterministic multi-objective optimization framework.

The first step in solving MOP problems is to obtain a payoff table. A payoff table contains single objective values from single optimization problem, and also provides potential ranges of the objectives (values of the upper,  $Z_L(j)$  and lower bounds  $Z_U(j)$ ). The minimum value of ( $Z_L$ ) of the Pareto surface is equal to the individual optimal value, while the maximum value ( $Z_U$ ) of the Pareto surface is the maximum value of that objective function found when minimizing other objectives. In this way, an approximate range of the right-hand-side  $L_k$  in the MINSOOP algorithm described previously is determined.

Fig. 3 shows the framework that utilizes the MINSOOP algorithm to address the deterministic MOP (no uncertainties) with six objective functions. Inputs are given to the multi-objective optimizer, which formulates single objective NLP optimization problems and passes them on to the NLP optimizer. The successive quadratic programming (SQP) method is used for this framework because it requires far fewer function and gradient evaluations than other methods for highly nonlinear constrained optimization, and it does not need feasible points at intermediate iterations. Both of these properties make SQP one of the most promising techniques for problems dealing with nonlinear constraint optimization, like process simulations [5]. The NLP optimizer gives the optimal value of the objective functions and the decision variables which are passed back to the multi-objective optimizer. This step is repeated about a 100–150 (estimated for MINSOOP algorithm) times until we get a good approximation of the Pareto optimal set of solutions to the problem.

#### 4. Deterministic MOP results and discussions

A payoff table is widely used in decision analysis, where it specifies the alternatives, acts, or events. Especially in MOP, a payoff table shows a potential range of values of each objective. Further, this helps to identify the approxi-

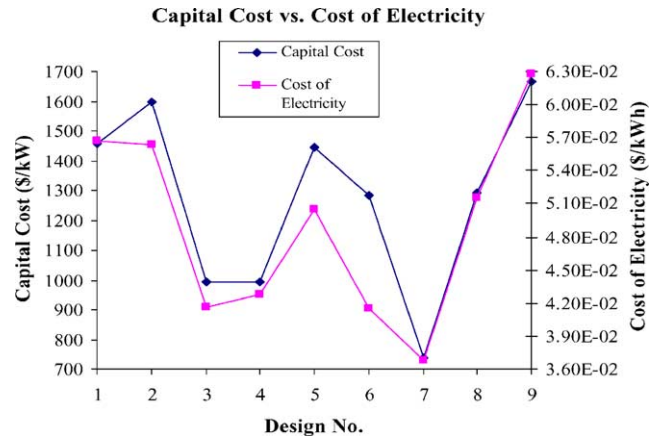


Fig. 4. Capital cost and cost of electricity for different SOFC-PEM hybrid plant designs.

mate trends of different objectives. The following subsection presents the analysis of payoff table for the deterministic MOP problem at hand.

##### 4.1. Payoff table analysis

The payoff table (Table 3) shows the objective values of 10 different designs that were obtained by maximizing and minimizing each of six objectives using the NLP optimizer. These values are plotted in Figs. 4–7.

To reduce the complexity of the problem, we can look at trends of various objectives plotted in Figs. 4–7. As we can see in Fig. 4, both capital cost and the cost of electricity follow the same trend. These two objectives are both maximized and minimized at the same set of decision variables. Therefore, we dropped one of the objectives and considered only one in the next stage of multi-objective optimization. Also in Fig. 5, the SOFC current density and CO<sub>2</sub> emissions are seen to have the same trend and can be combined as one. Although the value of SOFC current density shows a lot of variation the PEM current density remains relatively constant showing little sensitivity to the change in decision variables. Hence, we do not need to include it in the set of objectives. Fig. 6 indicates a similar trend between the PEM current density and the overall efficiency and Fig. 7 demonstrates that the capital cost, cost of electricity and CO<sub>2</sub> emissions follow the same trend. So finally, we carry three objectives (capital cost, overall efficiency and SOFC current density) to the next stage of multi-objective optimization. The three objectives were passed through the multi-objective optimization framework and MINSOOP to obtain the approximate Pareto set of solutions.

##### 4.2. Generating the Pareto set with MINSOOP

The reduced problem with only three objectives was then put into the multi-objective optimization framework as shown in Fig. 3. The capital cost was taken as the main objective with the other two objectives overall efficiency and

Table 3  
The bounds for different objectives calculated by deterministic multi-objective optimization: a deterministic payoff table

Design no.	Max ACEFF	Min ACEFF	Min CAP	Min COE	Max CO2EM	Min CO2EM	Max CDPEM	Min CDPEM	Min CDSOFC	Max CDSOFC
PWRTG	1475.1856	1465.51	1568.32587	1469.0955	1634.293405	1470.4051	1500.24142	1496.3793	1471.963	1471.99
ACEFF	0.7232446	0.52	0.6008321	0.58273	0.5426174	0.7066749	0.57530437	0.7077066	0.726258777	0.518409419
CAP	1456.4933	1599.80938	994.783579	993.38655	1444.397	1281.92286	739.9451	1289.6887	1664.6514	563.5081708
COE	5.67E-02	5.63E-02	4.16E-02	4.28E-02	5.05E-02	4.15E-02	3.68E-02	5.15E-02	6.28E-02	3.33E-02
CO2EM	0.2728659	0.37885684	0.3284591	0.3386584	0.36369784	0.2793638	0.343033	0.27885679	0.2717334	0.380681345
CDSOFC	101.86	672.009	531.131	616.0684	737.209689	157.6117844	678.179	149.0192	76.34382	873.9142717
CDPEM	290.3345	304.847	308.62476	307.62895	294.63637	294.38629	319.5917	287.22287	293.602876	318.9547032
UTIL	0.69993	0.55147	0.43617	0.40266	0.41114	0.7	0.4	0.7	0.7	0.4039
PPEM	23.3115	36.8556	39.7391	36.189	20.1208	26.3631	75	20	25.3265	72.12
ERAT	1.37434	5.69168	1.25	1.25	3.01218	1.79611	1.88651	1.70141	1.25	4.566
FUEL	20.1683	27.8188	25.8102	24.9279	29.7814	20.5743	25.7853	20.9073	20.0408	28.07
AIR	189.276	851.883	137.29	122.408	359.824	252.367	189.831	242.929	171.08	505.365

SOFC current density added to the problem as constraints. The reduced problem is shown below:

Min capital cost (CAP)  
 Subject to  
 Mass and energy balance constraints  
 Power rating (PWRTG) = 1472 kW  
 Overall efficiency (ACEFF)  $\leq \varepsilon_i, \quad i = 1, \dots, 100$  (3)  
 Current density SOFC (CDSOFC)  $\leq \chi_i, \quad i = 1, \dots, 100$   
 $ACEFF_{LB} \leq \varepsilon_i \leq ACEFF_{UB}$   
 $CDSOFC_{LB} \leq c_i \leq CDSOFC_{UB}$

Now the single objective optimization problems are solved with these additional constraints. The values of the bounds for the two are sampled by Hammersley sequence sampling. Each of the 100 iterations leading to the generation of the Pareto set is going to have a different combination of these bounds sampled between the actual upper and lower bounds of these objective functions.

4.3. Contour plots

The contour plots shown in Figs. 8 and 9 give a representation of the trade-off solutions. In Fig. 8, CO<sub>2</sub> emissions and overall efficiency are plotted on the two axes and the contours represent different values of capital cost required to obtain a design with the emissions and overall efficiency values at the corresponding point. Similarly in Fig. 9, the contours represent the capital cost required to obtain the corresponding values of overall efficiency and SOFC current density. With this analysis, we were able to obtain designs with up to 44% savings in capital cost, SOFC current density as high as up to 12 times and with up to 43% less CO<sub>2</sub> emissions than the base case. These plots represent the trade-off solutions and help in identifying several regions of operations that may not be evident intuitively.

Let us observe the contour plots carefully. The high efficiency and low emissions regions involve high capital costs. We do have some low cost regions at high SOFC current density but these involve low efficiency and high emissions. Another major low cost region is with SOFC current density between 350 and 500 mA/cm<sup>2</sup>, overall efficiency between 60 and 65% and CO<sub>2</sub> emissions between 0.30 and 0.32 kg/kWh. We see from Fig. 8 that it is possible to operate the plant at a low capital cost of less than US\$ 1100/kW and still get CO<sub>2</sub> emissions as low as 0.30 kg/kWh of electricity produced. We can see in Fig. 9 several regions where we have a moderate capital cost US\$ 1100–1200/kW and still get relatively good values of current density (300–700 mA/cm<sup>2</sup>).

The best part about this kind of representation is that given a particular value of current density or CO<sub>2</sub> emissions, we can easily identify the minimum cost, minimum emission or the maximum possible SOFC current density that we can achieve through this configuration. Then we can backtrack

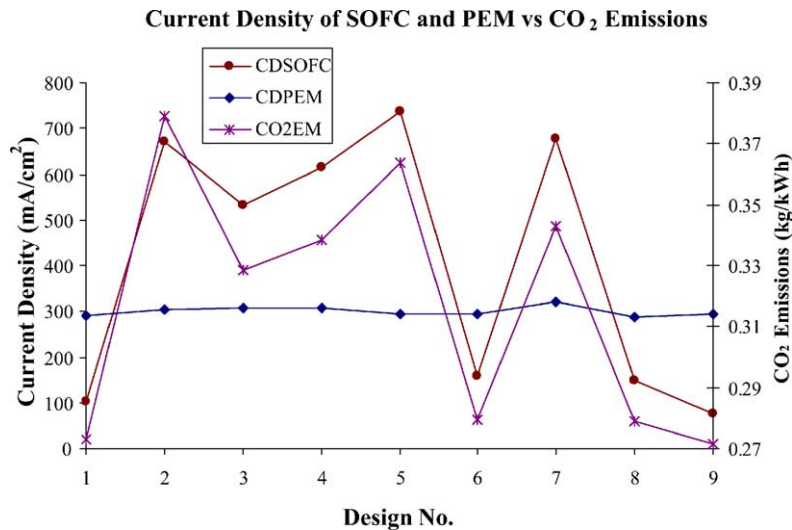


Fig. 5. Current density of SOFC, PEM and CO<sub>2</sub> emissions for different SOFC-PEM hybrid plant designs.

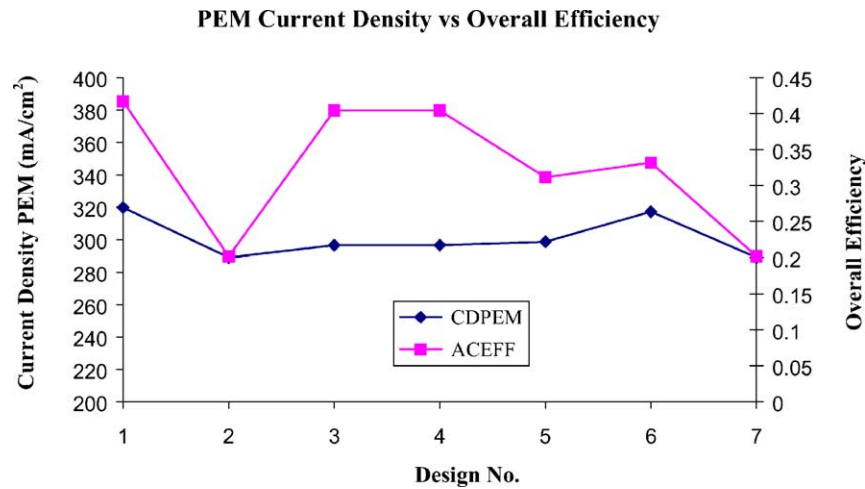


Fig. 6. Current density of PEM and overall efficiency for different SOFC-PEM hybrid plant designs.

and find the values of the decision variables where we need to operate to get this kind of performance. By doing this exercise just once we can also get an idea of the different amounts of capital cost involved and achievable current

densities in different geographical locations, as each location has different emission standards. Although this picture gives several insights into the current problem, it is far from a complete representation as we can only visualize three objectives out of the total seven considered.

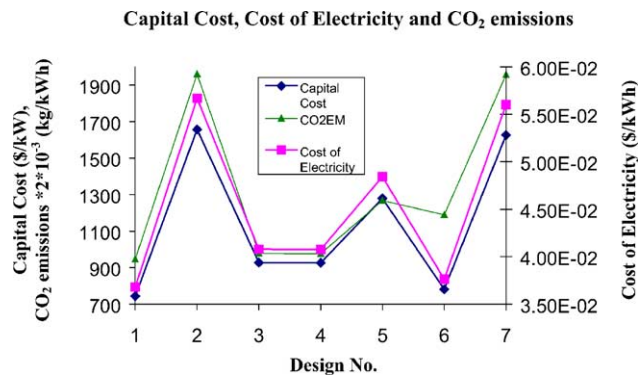


Fig. 7. Capital cost, cost of electricity and CO<sub>2</sub> emissions different SOFC-PEM hybrid plant designs.

#### 4.4. Normalized multiple objectives

For getting the complete trade-off representation we normalized all the objectives to get a value between 0 and 1, such that the lower the value of the normalized objective function the better the design.

If we want to maximize objective Z, then:

$$Z_{\text{normalized}} = \frac{Z_{\text{UB}} - Z}{Z_{\text{UB}} - Z_{\text{LB}}}$$

If we want to minimize objective Z, then:

$$Z_{\text{normalized}} = \frac{Z - Z_{\text{LB}}}{Z_{\text{UB}} - Z_{\text{LB}}}$$

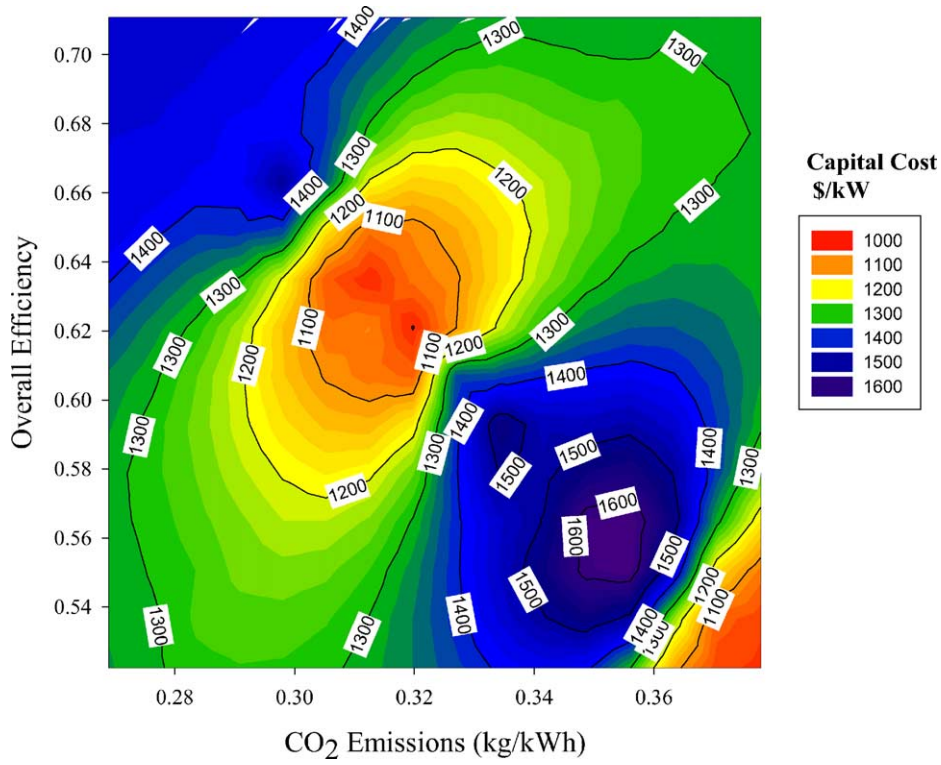


Fig. 8. Deterministic Pareto surface for CO<sub>2</sub> emissions, overall efficiency and capital cost.

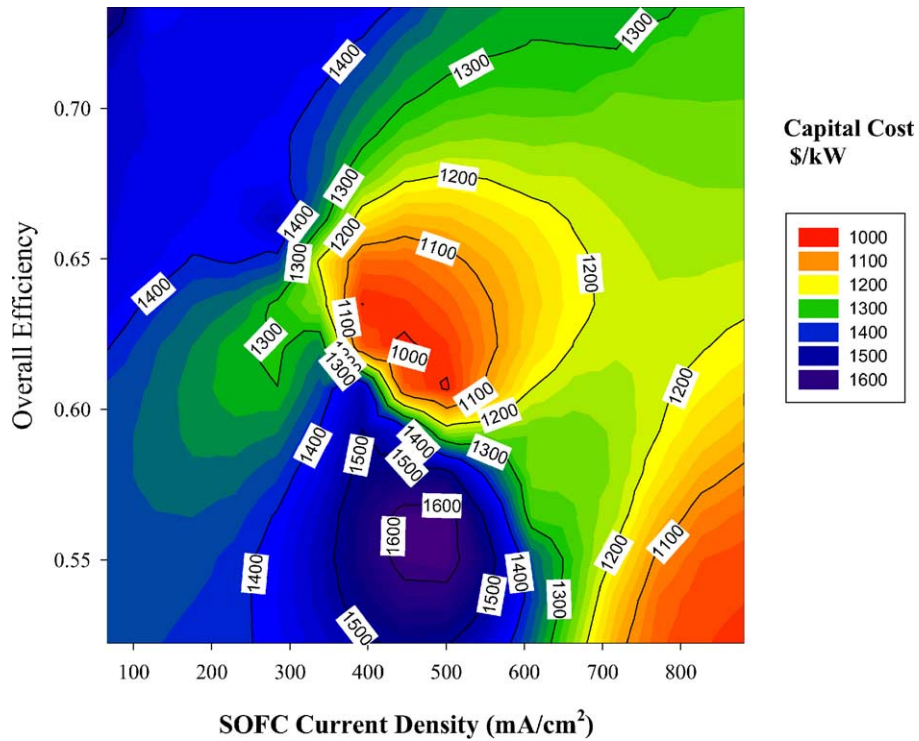


Fig. 9. Deterministic Pareto surface for overall efficiency, SOFC current density and capital cost.

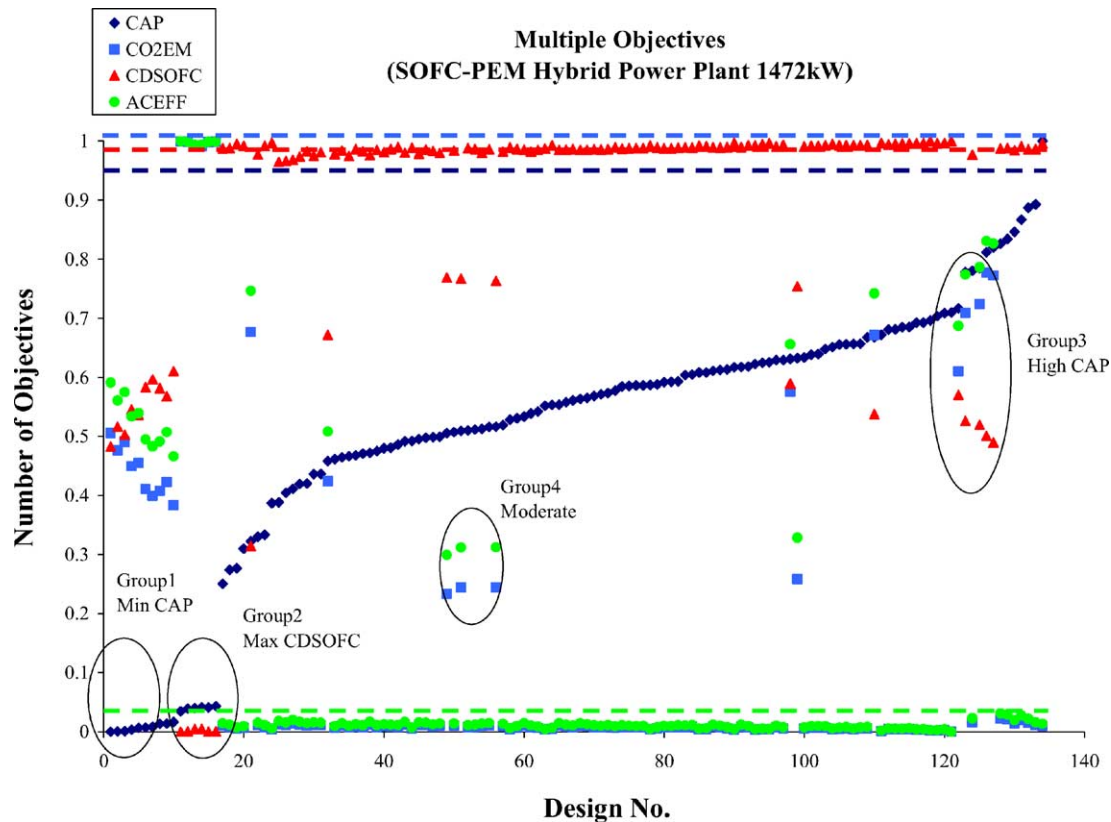


Fig. 10. Normalized objectives for design of SOFC-PEM hybrid fuel cell power plant.

These normalized values are plotted in Fig. 10. The dashed horizontal lines indicate the normalized values of the objectives corresponding to the base case. Here, we can identify several groups of designs with similar objective values. We have identified four such groups and presented their comparison with the base case. The corresponding objective functions and decision variable values for various groups and base case is presented in Table 4. Group 1 is the minimum capital cost groups with 44% lower capital cost than the base case. We also have lower CO<sub>2</sub> emissions (up to 34%) and higher SOFC current density (up to 6.5 times) than the base case. But we lose in the overall efficiency of the system.

Table 4  
Qualitatively similar design groups for the deterministic case

	Base case	Group 1 Min CAP COST	Group 2 Max CDSOFC	Group 3 Max CAP COST
CAP	1773	986–1001	1014–1022	1501–1657
CO2EM	0.468	0.31–0.324	0.376–0.378	0.332–0.354
CDSOFC	75	384–487	876–881	401–483
ACEFF	0.73	0.6–0.64	0.522–0.524	0.558–0.595
UTIL	0.7	0.45–0.51	0.4	0.68–0.7
PPEM	25	31–36	74–75	22–37
ERAT	1.25	1.25	6	4.65–5.65
FUEL	20	22–24	25–28	23–27
AIR	200	133–143	583–630	762–1001

These types of designs are recommended when cost is the most important factor. Group 2 presents designs with maximum SOFC current density (up to 12 times that of base case) and low capital cost (1014–1022 \$/kW). But again we have to compromise heavily on efficiency and CO<sub>2</sub> emissions (22% higher than Group 1 though still 20% lower than base case). These designs are recommended when emission standards are not that stringent and high SOFC density is desired. These also require operation of PEM at a higher pressure. Group 3 designs are the ones representing the high cost regions right in the middle of the Pareto surface plots. These have higher cost than Groups 1 and 2 though still up to 16% lower than base case. These also have intermediate values of current density, CO<sub>2</sub> emissions and overall efficiency. These regions in the Pareto surface should be avoided. Group 4 designs have efficiency on the higher side and lower values for SOFC current density and CO<sub>2</sub> emissions. The capital cost is also in the intermediate range. A decision maker might want to choose such a design if he/she wants the process to run at “moderate” conditions.

## 5. Effect of uncertainties on the MOP trade-off surfaces

The earlier results presented the MOP trade-off surfaces for the conceptual design when uncertainties are not



considered. Diwekar [11] described three kinds of uncertainties featured in the life cycle of a plant:

1. Uncertainty with respect to the model parameters: These parameters are a part of the deterministic model and not actually subject to randomness. Theoretically their value is an exact number. The uncertainty results from the impossibility of exactly modeling the physical behavior of the system.
2. Uncertainty in the input variables: This kind of uncertainty originates from the random nature and unpredictability of certain process inputs.
3. Uncertainty in the initial conditions: These types of uncertainties result due to the complications in predicting the initial conditions of the operation.

As stated earlier, this technology is new and is at a conceptual stage, therefore, we are considering the first type of uncertainties, i.e. uncertainties related to model parameters. Specially, we are concentrating on the two fuel cell models in this study.

### 5.1. Characterizing and quantifying uncertainties

In general, an essential component (apart from the electrochemical reactions) of a fuel cell model is the current density characteristic of a particular fuel cell. The current density characteristic provides the voltage and current density profile, and is a function of fuel cell design. In this work, we have used the experimental data reported in the literature (for SOFC [12] and for PEM [12]) to characterize uncertainties in the current density characteristic. A new model parameter called uncertainty factor (UF) is defined as the ratio between the experimental current density to that calculated by the model. After calculation of uncertainty factor for each of the current density data, the next step is the quantification of uncertainty. The values of  $UF_{SOFC}$  and  $UF_{PEM}$  are fitted to a probability density function (PDF). This PDF gives the probability or frequency of occurrence of each uncertainty factor. Figs. 11 and 12 show how we characterized the SOFC and PEM current density uncertainty factors. As is clear from the graph (Fig. 11), the distribution of  $UF_{SOFC}$  is triangular and the most likely value is skewed to the right.

**PDF for Uncertainty Factor in CDSOFC**

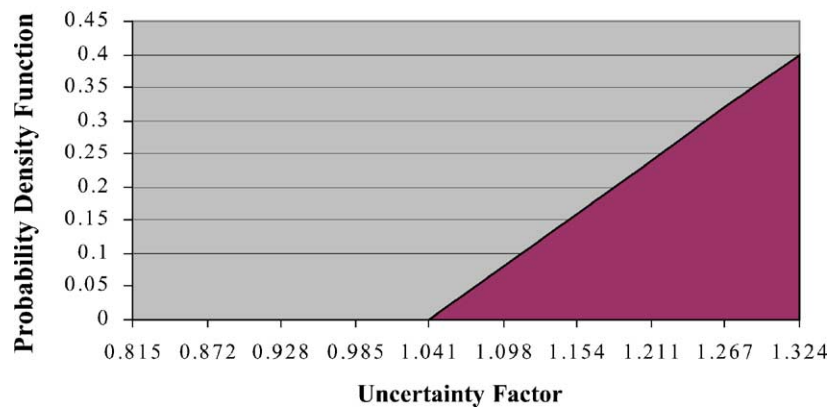


Fig. 11. Uncertainty factor distribution for SOFC current density.

**PDF for Uncertainty Factor in CDPEM**

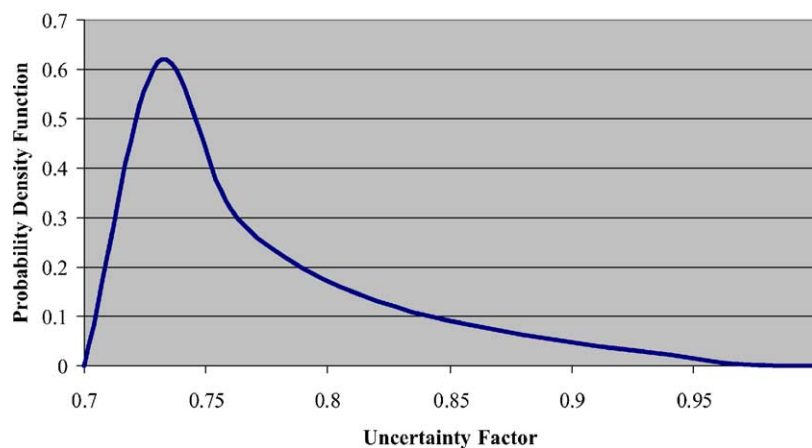


Fig. 12. Uncertainty factor distribution for PEM current density.

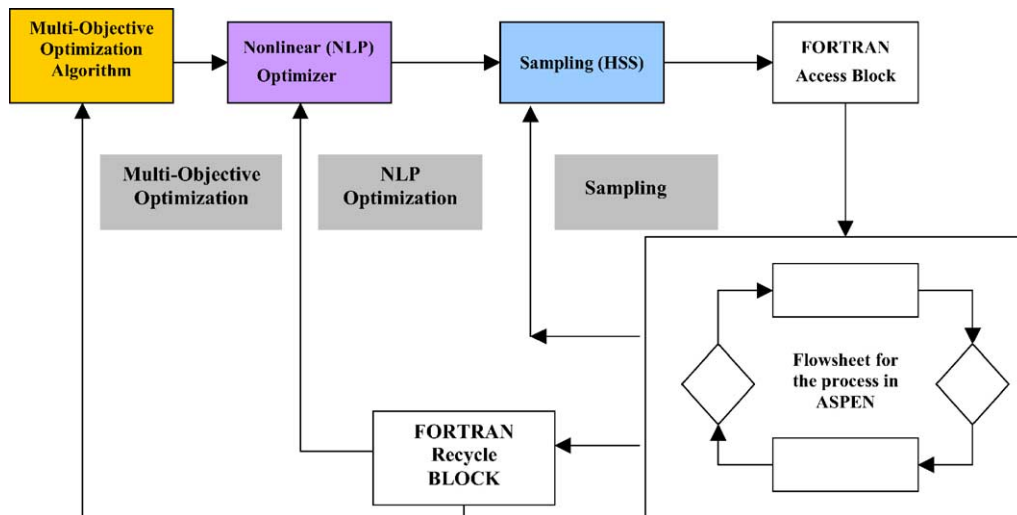


Fig. 13. Schematic of the propagation of uncertainty through the ASPEN model.

The least likely value is 0.928 and the most likely value is 1.3235. Similarly a log-normal distribution (Fig. 12) was obtained for the PEM current density uncertainty factor.

5.2. The MOP framework under uncertainty

Once probability distributions are assigned to the uncertain parameters, the next step is to propagate the uncertainties and obtain the stochastic multi-objective optimization tradeoff surfaces. The conceptual framework for this stochastic MOP problem is shown in Fig. 14 where the deterministic model in Fig. 3 is replaced by a stochastic modeling framework with a sampling loop.

The input of objective functions and decision variables are passed to the multi-objective optimizer. The multi-objective optimizer converts the multi-objective to a single objective problem using MINSOOP [7]. The single objective function and decision variables are passed to the NLP optimizer which runs through the model via the efficient Hammer-sley sequence sampling [9,10] and passes the results in terms of probabilistic objectives and constraints back to the multi-objective optimizer. At the end of all the runs, we get the Pareto set of MOP solutions under uncertainty from the multi-objective optimizer.

Fig. 13 shows the ASPEN Plus [13,14] framework developed for the algorithmic framework described in Fig. 14. The

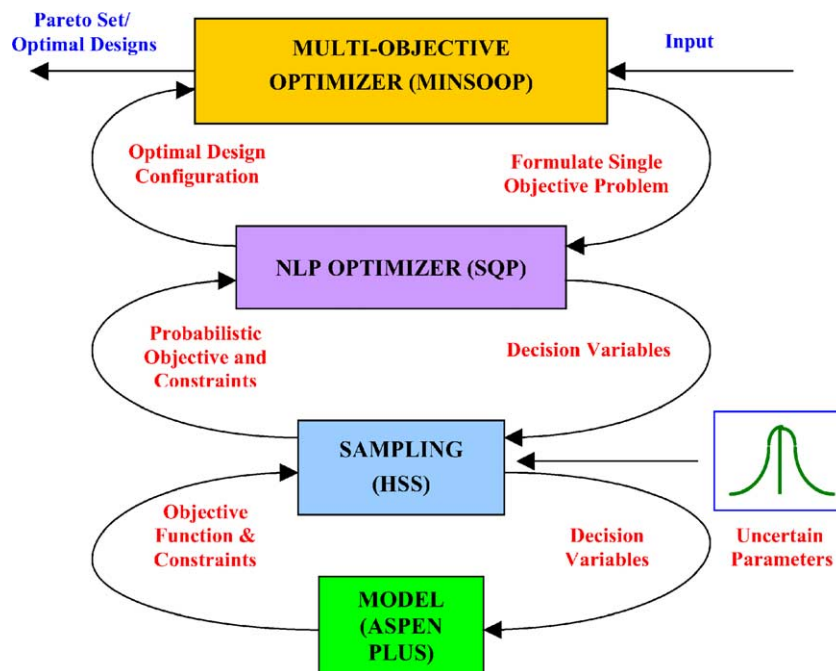


Fig. 14. Stochastic multi-objective optimization framework.

Table 5  
The bounds for different objectives calculated by stochastic multi-objective optimization: a stochastic payoff table

Design no.	1	2	3	4	5	6	7	8	9	10
PWRTG	1469.0811	1503.85904	1487.78599	1550.2675	1480.164724	1474.446793	1472.66539	1483.29328	1468.19915	1471.686032
ACEFF	0.7199859	0.53642795	0.59571748	0.5783063	0.493840626	0.721052248	0.57419628	0.7103846	0.724465776	0.51601781
CAP	1679.2573	1529.93233	975.530499	978.95759	1426.286958	1676.189157	733.659472	1371.77014	2256.227124	464.5987811
COE	6.28E-02	5.35E-02	4.19E-02	4.10E-02	5.16E-02	6.26E-02	3.66E-02	5.38E-02	7.53E-02	3.07E-02
CO2EM	0.2686463	0.36057322	0.32468672	0.3344621	0.391667966	0.268249012	0.33685616	0.27227723	0.26698508	0.374835034
CDSOFC	85.243239	866.174172	596.892156	704.88103	998.8307174	86.12588685	759.92657	134.962445	85.97010126	1002.303931
CDPEM	241.99771	240.641451	249.644838	251.08606	242.2873382	244.7935206	261.006594	234.533736	253.0061513	260.3966995
CONV	0.6991791	0.4	0.44226414	0.4023899	0.426851368	0.7	0.4	0.7	0.7	0.4
PPEM	29.239233	20	33.8554951	35.880635	22.60108345	35.44742338	75	20	64.85995748	71.48
ERAT	1.25	2.80530394	1.25	1.25	5.208278559	1.25	1.68682003	1.45011707	1.25	4.32
FUEL	20.175789	27.7207186	24.6949911	26.506811	29.63684651	20.21953321	25.3601874	20.6463007	20.03899098	28.2
AIR	172.03036	303.473332	133.191574	130.07407	642.8046466	172.6057713	166.938818	204.465241	171.0645572	475.5361

MINSOOP based MOP algorithm block formulates a single objective optimization problem to obtain the Pareto set. The ASPEN model contains detailed information for the process and predicts deterministic values of the objective functions and constraints. The NLP and the STOCHA blocks generate the values of decision variables and uncertain parameters. The Fortran access block accesses the uncertain variables and decision variables of the process and replaces them with the values obtained by the NLP and the STOCHA blocks. The Fortran recycle block makes sure that the model calculations and stochastic calculations are repeated till the NLP optimizer achieves convergence for all the single objective optimization problems generated by the MOP block.

5.3. Stochastic MOP analysis

The payoff table is obtained by performing the optimization for each stochastic objective (expected value in this case) without any constraints thereby obtaining the bounds for each objective. These values are shown in Table 5. The Pareto surfaces obtained as a result of the stochastic optimization framework are shown in Figs. 15–16. Note that there is a considerable difference in the contour shape and levels between the stochastic and deterministic Pareto surfaces. This can be attributed to the fact that the optimum decision variables are different for the two cases as is evident from Tables 3 and 5. For example, for the maximum overall efficiency case, airflow is lower for stochastic case than the deterministic case. Minimum efficiency designs have higher current density for stochastic case and lower air flow. This decreased the capital cost more in the stochastic case than in the deterministic case which changed the trade-off surface. Similarly in case of maximum CO<sub>2</sub> emission designs, the airflow in the stochastic case is almost doubled resulting in increased the capital cost and thereby changing the Pareto surface (Fig. 15).

In Fig. 15 capital costs have shifted towards lower levels as compared to the deterministic designs shown in Fig. 8. The high cost regions at the lower right area of the deterministic Pareto surface has shrunk remarkably and has split into two small high cost area. The highest cost regions in the upper right corner has disappeared altogether and has resulted in moderate cost regions of US\$ 1100–1200/kWh. In case of Fig. 16, the high cost regions at the right hand side region of the deterministic surface has disappeared and has been replaced by low cost regions of US\$ 900–1000/kWh. We can see two high cost regions at the lower right area of the stochastic surface. Good operating regions can be identified at the centre of the plot with lowest cost of around US\$ 800/kWh, moderate current density of 400–500 mA/cm<sup>2</sup> and overall efficiency of 0.62–0.64.

To analyze the results further, we changed the contour plots 9 and 16 to Figs. 17 and 18, respectively. Figs. 17–18 show the comparison between deterministic and stochastic surfaces of overall efficiency, capital cost (on the two axes) and SOFC current density as the contours. Note that

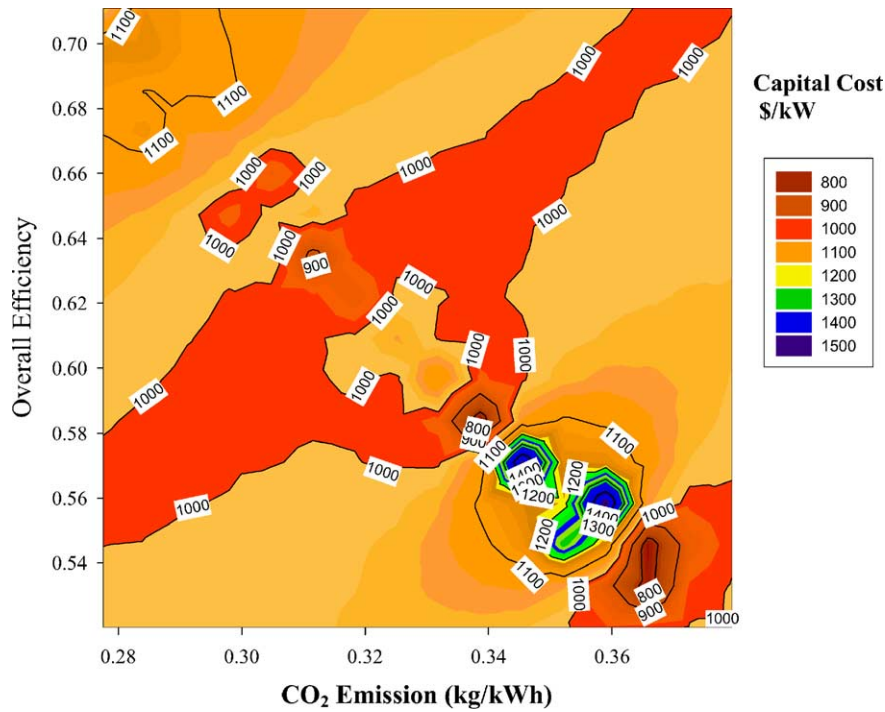


Fig. 15. Stochastic Pareto surface for overall efficiency, CO<sub>2</sub> emissions and capital cost.

the most likely value of the SOFC current density  $UF_{SOFC}$  was 1.3235. We see that the most likely value for the optimal current density in the Pareto surface is 300 mA/cm<sup>2</sup> in the center region of the deterministic surface. However, the most likely value in the stochastic surface appears to be 600–700 mA/cm<sup>2</sup>, not corresponding to the most likely

value of the most influential uncertain parameter  $UF_{SOFC}$ . This can be attributed to the nonlinearities of the model and also emphasizes the need to consider uncertainty analysis.

We also obtained qualitatively similar designs using normalized objectives similar to the deterministic case. Table 6

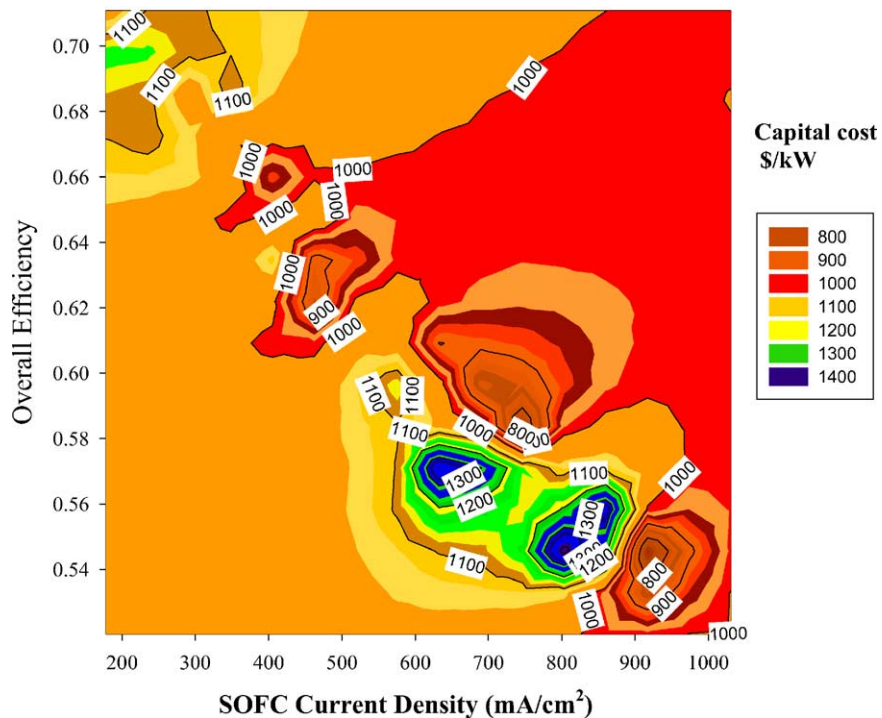


Fig. 16. Stochastic Pareto surface for SOFC current density, overall efficiency and capital cost.

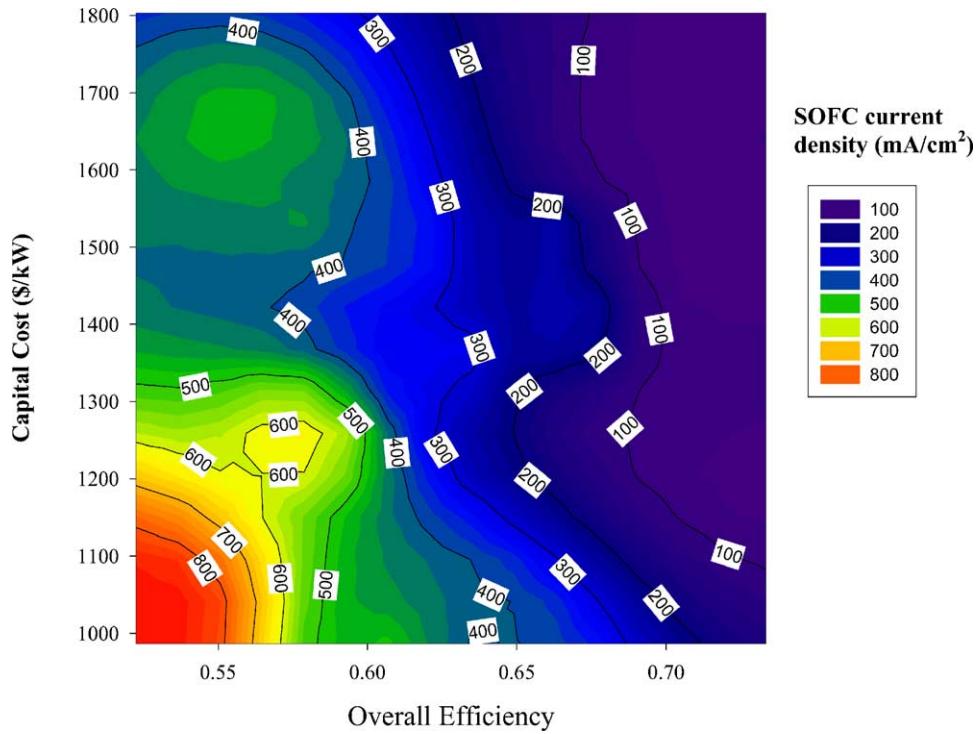


Fig. 17. Deterministic Pareto surface for overall efficiency, capital cost and SOFC current density.

shows the comparison between deterministic and stochastic groups for three types of designs identified earlier, namely, min capital cost (Group 1), max current density (Group 2) and max capital cost (Group 3). It looks like

inclusion of uncertainties in the current density characteristics have increased the range of decision variables for almost all the groups, thereby providing more flexibility to designer.

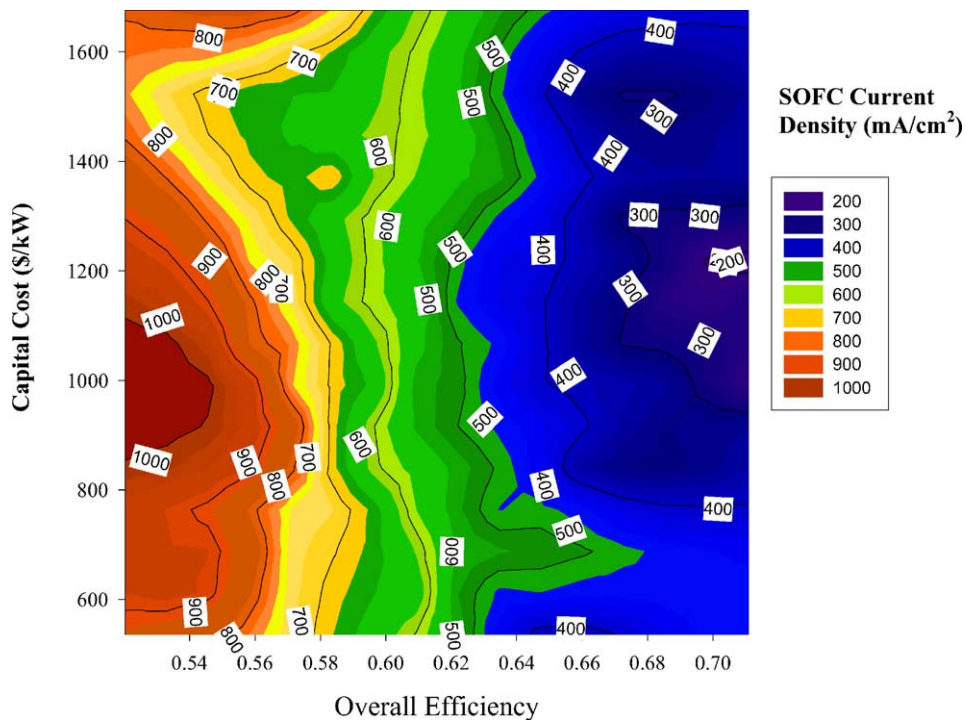


Fig. 18. Stochastic Pareto surface for overall efficiency, capital cost and SOFC current density.

Table 6  
Comparison of qualitatively similar design groups for stochastic and deterministic cases

	Base case	Deterministic			Stochastic		
		Group 1 Min CAP COST	Group 2 Max CDSOFC	Group 3 Max CAP COST	Group 1 Min CAP COST	Group 2 Max CDSOFC	Group 3 Max CAP COST
CAP	1773	986–1001	1014–1022	1501–1657	610–787	981–1020	1340–1675
CO2EM	0.468	0.31–0.324	0.376–0.378	0.332–0.354	0.31–0.36	0.376–0.379	0.29–0.35
CDSOFC	75	384–487	876–881	401–483	472–946	1015–1030	291–828
ACEFF	0.73	0.6–0.64	0.522–0.524	0.558–0.595	0.537–0.634	0.52–0.53	0.55–0.62
UTIL	0.7	0.45–0.51	0.4	0.68–0.7	0.4–0.59	0.4	0.4–0.6
PPEM	25	31–36	74–75	22–37	71–75	74.7–75	20–62
ERAT	1.25	1.25	6	4.65–5.65	1.25–3.58	6	1.25–3.89
FUEL	20	22–24	25–28	23–27	15.64–36.83	20.77–44.53	20.44–29.19
AIR	200	133–143	583–630	762–1001	85.89–367.26	595–1050	130.1–439

## 6. Conclusion

We have shown in this paper that the SOFC-PEM hybrid plant design is a multi-objective optimization problem and presented a framework to handle these problems deterministically. The MOP framework presented here allows for an efficient determination of the Pareto surface for the problem. The Pareto surface provides an effective means to assess the trade-offs amongst the multiple objectives. We have shown that a multi-objective optimization framework can help in identifying designs, which are more cost effective as well as environmentally friendly. The paper also presents a methodology to characterize and quantify uncertainties in fuel cell models. The effects of uncertainty on the objectives have been analyzed by comparing the stochastic and deterministic trade-off surfaces of different objectives. We found that uncertainty had a considerable effect on the objectives and the trade-off surfaces were markedly different. Stochastic MOP identified a different decision variable space in which to operate, with a lower capital cost. This paper provides a first step towards an integrated approach to synthesis of new and more efficient fuel cell hybrid power plants.

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